Tutorial 1

Engineering Mathematics 214

Topic: Exact Equations (Section 2.4 in Zill)

Problem 1. *For each of the following differential equations, determine whether or not it is exact, and if so solve it.*

(a) $(2x - 1)\mathrm{d}x + (3y + 7)\mathrm{d}y = 0$

(b) $y\cos(x)\mathrm{d}x + x\sin(y)\mathrm{d}y = 0$

(c) $(e^x \sin(y) - 2y\sin(x))\mathrm{d}x + (e^x\cos(y) + 2\cos(x))\mathrm{d}y = 0$

Problem 2. *Which of the following functions is an integrating factor for the DE below*

$$x^2y^3\,\mathrm{d}x - x(1 + y^2)\mathrm{d}y = 0.$$

(a) xy

(b) xy^2

(c) $\frac{1}{xy^2}$

(d) $\frac{1}{xy^3}$

(e) None of the above are integrating factors.

Problem 3. *Solve the following initial value problems:*

(a) $(e^x + y)\mathrm{d}x + (2 + x + ye^y)\mathrm{d}y = 0, \qquad y(0) = 1$

(b) $\left(\frac{3y^2 - t^2}{y^5}\right)\frac{\mathrm{d}y}{\mathrm{d}t} + \frac{t}{2y^4} = 0, \quad y(1) = 1$

Problem 4. *Find the value of b for which the equations below are exact, and then solve them for that value of b.*

(a) $(xy^2 + bx^2y)\mathrm{d}x + (x + y)x^2\,\mathrm{d}y = 0$

(b) $(ye^{2xy} + x)\mathrm{d}x + bxe^{2xy}\,\mathrm{d}y = 0$

Problem 5. *Solve the following differential equations by finding appropriate integrating factors:*

(a) $y(x + y + 1)\,\mathrm{d}x + (x + 2y)\,\mathrm{d}y = 0$

(b) $\cos x\,\mathrm{d}x + \left(1 + \frac{2}{y}\right)\sin x\,\mathrm{d}y = 0$

(c) $(3xy + y^2) + (x^2 + xy)\frac{\mathrm{d}y}{\mathrm{d}x} = 0$