B.Sc. Honours in Mathematics 2023

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# 1 Practical Information

#### 1.1 Stellenbosch University and Department of Mathematical Sciences

Stellenbosch University is situated in a picturesque wine growing region nestled in the mountains, approximately 50km from Cape Town. Mathematics forms one division of the Department of Mathematical Sciences. (The other divisions are Applied Mathematics and Computer Science.) Our research interests in Mathematics are reflected in the optional modules that form part of the B.Sc. Honours degree curriculum. On successful completion of the Honours degree, further study towards Masters and Ph.D. degrees in Mathematics is possible.

## 1.2 Degree structure

The normal duration of the B.Sc. Honours degree in Mathematics is one year, studying full time. In exceptional circumstances and at the discretion of the Department of Mathematical Sciences, it can be extended to two years.

Students must complete modules totaling 128 credits towards the degree, with 64 credits in the first semester and 64 credits in the second semester. (Details of some of the available modules are given in Sections 3 and 4 below.) One of the modules takes the form of a research project of the student's choice.

The programme for each student will be arranged to accommodate the student's background and interests. Subject to the Department's approval, a maximum of half of the degree credits may be taken in other divisions of the Department or in other University departments.

Due to departmental expertise and the career and research opportunities they provide, the following possible focuses are suggested:

- Mathematics and
- *Biomathematics*.

The suggested curricula for these focus areas are given below.

#### 1.3 Requirements for admission

The admission requirements are as follows.

- A BSc degree with Mathematics as major or an equivalent qualification.
- $\bullet$  A final mark of at least 60% for Mathematics 3.

• For the Biomathematics focus, you must have a BSc degree approved by the Biomathematics programme committee with an average mark of at least 60% for the relevant third-year modules.

Note that meeting the basic admission requirements does not guarantee acceptance. Each application is carefully considered and applicants may be requested to attend an interview which could be conducted online.

Stellenbosch University is a multilingual university. At graduate level the language of instruction (Afrikaans and/or English) is in general determined by the preferences of the students and the abilities of the lecturers. Proficiency in Afrikaans is not a prerequisite for admission to the Honours programme, but academic competence in English is necessary.

#### 1.4 Requirements to obtain the degree

In order to obtain the B.Sc. Honours degree in Mathematics, a student must achieve at least 50% in every module in his or her approved programme.

If a student fails a theory-based module, he or she may apply to repeat this module in the following year. Application can be made to repeat a maximum of two such modules. Admission to the relevant module(s) in the following year is solely at the discretion of the department. However, the Honours project module cannot be repeated and if a student fails this, he or she will not graduate with the B.Sc. Honours degree in Mathematics.

## 1.5 Facilities

All students have access to the excellent facilities of Stellenbosch University. There are shared computers with e-mail and internet access and students have access to the well equipped University library.

## 1.6 Financial Support

All eligible graduate students are encouraged to apply for bursaries through the University as well as the National Research Foundation. Additional income can sometimes be earned by being employed on a part-time basis as a tutor for undergraduate mathematics modules. Details about application procedures can be obtained from the head of the Department or the secretary of the Mathematics Division.

## 1.7 Contact Information

The head of the Department of Mathematical Sciences is Prof. I.M. Rewitzky (rewitzky@sun.ac.za), and the head of the Mathematics Division is Prof. L. van Wyk (lvw@sun.ac.za). The convenor of the pure mathematics focus of Mathematics Honours is Prof. G. Boxall (gboxall@sun.ac.za). The convenor of the biomathematics focus of Mathematics Honours is Prof. C. Hui (chui@sun.ac.za). The secretary of the Mathematics Division is Mrs. L. Muller (lisam@sun.ac.za). The postal address for the Mathematics Division is:

Department of Mathematical Sciences Tel: (021) 808-3282 Mathematics Division University of Stellenbosch Private Bag X1 Matieland 7602 South Africa

## 2 Suggested Focus Areas for the Degree

The Mathematics Honours programme is flexible, and exact module choices will be decided upon in consultation with individual students. The choice of modules should give a coherent focus to the programme, leading to opportunities for further study and employment. Suggested curricula are outlined below.

## 2.1 Mathematics

This focus is for students wanting a rigorous mathematics education. It consists principally of modules taught in the Mathematics Division and is usually followed by students who have a love for "pure mathematics", in particular those who intend to follow a career in research and/or teaching. (The number of credits of each module is given in brackets.)

First Semester	Second Semester
Algebra (16)	four 8-credit Choice Modules
Functional Analysis and Measure Theory (16)	subject to departmental approval
Real and Complex Analysis (16)	Honours Project (32)
Set Theory and Topology (16)	

## 2.2 Biomathematics

Biomathematics is a multidisciplinary research field at the interface of mathematics and biosciences. It aims to explain and predict the dynamics, structure and function of biological systems and help develop appropriate interventions or treatment strategies. It covers a wide range of research areas, including brain and neuroscience; cancer growth and treatment; cell biology; developmental biology; ecology and evolution; infectious and non-infectious diseases; mathematical, computational, biophysical and statistical modelling; microbiology; molecular biology; biochemistry; networks and complex systems; physiology; pharmacodynamics; behaviour and game theory; to name a few. (The number of credits of each module is given in brackets).

First Semester	Second Semester
Computational And Discrete Methods	Honours project (32)
in Biomathematics (16)	
Non-linear Dynamical Systems	Advanced Topics in Biomathematics III (16)
in Biomathematics (16)	
Advanced Topics in Biomathematics I (8)	Advanced Topics in Biomathematics IV (8)
Advanced Topics in Biomathematics II (8)	Choice module (8)
Selected Topics from Biological Sciences (8)	
Selected Topics from Biomedical Sciences (8)	

Students registering for this focus will spend the first half of the year (January-June) at AIMS-SA (African Institute for Mathematical Sciences, Muizenberg), where they will attend a number of special modules presented by local and international scientists on topics in biomathematics and bio-informatics. In the second half of the year the students will be based at Stellenbosch University working with their supervisors on specific projects.

In the remainder of this document we give further details on modules and projects suitable for the Mathematics focus of Mathematics Honours.

## 3 First Semester Modules for Mathematics Focus

The modules offered in the first semester are the core modules for the programme. Each module is worth 16 credits.

## 3.1 Algebra (711)

The first and second quarters are dedicated to group theory and Galois theory, respectively.

In the group theory course we will introduce basic notions such as conjugates, normalisers and normal subgroups, after which we will treat various examples, such as the circle group, dihedral groups and the quaternions. (The additive group of integers modulo n and other cyclic groups are already known from 3rd year courses). Other topics include the conjugate class equation of a group, p-groups, Cauchy's Theorem and the Sylow Theorems. The Galois theory course builds upon the field theory from the 3rd year algebra course. This theory arose from investigating solutions to polynomial equations, and combines central themes from classical and modern algebra. It is closely linked with the theory of solvable groups, and some of the greatest mathematicians of the last 200 years have contributed to this subject.

Requirements: A third year course in basic algebra (Mathematics 314).

**Textbook:** Notes will be provided.

Lecturers: Prof K-T Howell and Dr D Basson

#### 3.2 Functional Analysis and Measure Theory (712)

The first quarter is dedicated to functional analysis and the second quarter to measure theory.

Functional Analysis: Metric and Banach spaces, bounded linear operators, functionals and dual spaces. Introduction to Hilbert spaces. The Hahn Banach theorem and its consequences, the Baire category theorem, the uniform boundedness theorem.

Measure Theory: Lebesgue outer measure, measurable set and measure, measurable functions, Littlewood's Principles. Shortcomings of the Riemann integral, the Lebesgue integral and convergence theorems. The  $L^p$  spaces.

**Requirements:** A third year course in metric spaces or real analysis (Mathematics 365).

#### Textbooks:

Functional Analysis: E. Kreyszig: Introductory Functional Analysis with Applications, John Wiley & Sons Inc., New York, 1978.

Measure Theory: H. L. Royden: Real Analysis, Macmillan Publishing Co., Inc., New York, 1968.

Lecturers: Dr R Benjamin and Dr R Heymann

## 3.3 Real and Complex Analysis (713)

This course is a continuation of the third year course in complex analysis and involves, among others, the following topics: Harmonic functions, Jensen's formula, Weierstrass products, the Riemann mapping theorem and the gamma and zeta functions.

Requirements: Third year courses in complex analysis (Mathematics 324) and in real analysis (Mathematics 365).

Textbook: Notes will be provided.

Lecturer: Prof G Boxall and Dr D Ralaivaosaona

#### 3.4 Set Theory and Topology (714)

In this course, each student will be required to complete assignments from one (or more) of the following areas of mathematics: axiomatic set theory (Zermelo-Frankel axioms, Zorn's lemma and the well ordering principle, cardinal and ordinal arithmetic), general topology (topology via neighborhoods, closure and interior, compactness, separation axioms, continuous functions and homeomorphisms), duality theory (lattices and Boolean algebras, Stone, Birkhoff and Priestly dualities), algebraic topology (homotopy of paths, definition and computation of fundamental group/groupoid of a topological space), and categorical topology (basic topological constructions viewed as limits and colimits in the category of topological spaces, topological functors). Students with a background in some of these areas from their undergraduate studies will be required to complete assignments that complement their background.

**Requirements:** A third year course in real analysis (Mathematics 365) or the module entitled "Foundations of Abstract Mathematics II" (Mathematics 378).

**Textbook:** Appropriate texts to be decided with individual students.

Lecturers: Dr J Gray

## 4 Second Semester Choice Modules for Mathematics Focus

We have a large list of 8-credit modules available in the second semester and each student chooses four of these (or possibly fewer if they are taking approved modules from other departments or divisions). Please note that some of

the modules listed below may not be available every year.

Algebraic Number Theory (747) Computational Algebra (748) Wavelet analysis (749) Functional Analysis II (751) Measure Theory II (752) Category Theory (753) Logic (754)Concrete Mathematics (755) Topics in Algebra (756) Complex Analysis II (757) Advanced Analysis (760) Advanced Abstract Algebra (761) Number theory (762)Advanced Combinatorics (767) Algebraic Curves (768) Algebraic Geometry (769) Asymptotic Methods (771) Categorical Algebra (772) Differential Geometry (773) Functional Analysis III (774) Hilbert Spaces and  $C^*$ -algebras (775) Knot Theory (776) Lie Groups and Lie Algebras (780) Model Theory (784) Operator Theory (785) Universal Algebra (781) Representation Theory (782)Analytic Number Theory (783)

# 5 Honours Project (746) for Mathematics Focus

Students have to complete a research project on a topic of their choice. This will be evaluated through a written report and an oral presentation. The project is worth 32 credits and it is expected that most of the work on it will be done in the second semester.