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B.Sc. Honours

in

Mathematics

2020

Contents

1 Practical Information

1.1 Stellenbosch University and Department of Mathematical Sciences

Stellenbosch University is situated in a picturesque wine growing region nestled in the mountains, approximately 50km from Cape Town. Mathematics forms one division of the Department of Mathematical Sciences. (The other divisions are Applied Mathematics and Computer Science.) Our research interests in Mathematics are reflected in the optional modules that form part of the B.Sc. Honours degree curriculum. On successful completion of the Honours degree, further study towards Masters and Ph.D. degrees in Mathematics is possible.

1.2 Degree structure

The normal duration of the B.Sc. Honours degree in Mathematics is one year, studying full time. In exceptional circumstances and at the discretion of the Department of Mathematical Sciences, it can be extended to two years.

Students must complete modules totaling 128 credits towards the degree, with 64 credits in the first semester and 64 credits in the second semester. (Details of some of the available modules are given in Sections 3 and 4 below.) One of the modules takes the form of a research project of the student's choice. The standard lecture load is three hours a week for a 16 credit module and two hours a week for an 8 credit module. However, additional tutorials may be scheduled and a lot of work is expected of students outside of lectures.

The programme for each student will be arranged to accommodate the student's background and interests. Subject to the Department's approval, a maximum of half of the degree credits may be taken in other divisions of the Department or in other University departments. The guiding principle is the formation of a coherent, well-focused curriculum.

Due to departmental expertise and the career and research opportunities they provide, the following possible focuses are suggested:

- *Mathematics and*
- *Biomathematics.*

The suggested curricula for these focus areas are given below.

1.3 Requirements for admission

A B.Sc. degree with Mathematics as major subject or an equivalent qualification is needed to gain entrance to the Mathematics Honours programme. A mark of at least 60% for Mathematics 3 is required. Note that meeting these basic requirements does not guarantee acceptance.

Stellenbosch University is a multilingual university. At graduate level the language of instruction (Afrikaans and/or English) is in general determined by the preferences of the students and the abilities of the lecturers. Proficiency in Afrikaans is not a prerequisite for admission to the Honours programme, but academic competence in English is necessary.

1.4 Requirements to obtain the degree

In order to obtain the B.Sc. Honours degree in Mathematics, a student must achieve at least 50% in every module in his or her approved programme.

If a student fails a theory-based module, he or she may apply to repeat this module in the following year. Application can be made to repeat a maximum of two such modules. Admission to the relevant module(s) in the following year is solely at the discretion of the Department of Mathematical Sciences. However, the Honours project module cannot be repeated and if a student fails this, he or she will not graduate with the B.Sc. Honours degree in Mathematics.

1.5 Facilities

All students have access to the excellent facilities of Stellenbosch University. There are shared computers with e-mail and internet access and students have access to the well equipped University library.

1.6 Financial Support

All eligible graduate students are encouraged to apply for bursaries through the University as well as the National Research Foundation. Additional income can be earned by being employed on a part-time basis as a tutor for undergraduate mathematics modules. Details about application procedures can be obtained from the head of the Department or the secretary of the Mathematics Division.

1.7 Contact Information

The head of the Department of Mathematical Sciences is Prof. I.M. Rewitzky (rewitzky@sun.ac.za), and the head of the Mathematics Division is Prof. L. van Wyk (lvw@sun.ac.za). The Mathematics Honours Coordinator is Dr G. Boxall (gboxall@sun.ac.za), except in the first semester of 2020 when it will be Dr D. Basson (djbasson@sun.ac.za). The convenor of the Biomathematics Focus is Prof. I.M. Rewitzky (rewitzky@sun.ac.za). The secretary of the Mathematics Division is Mrs. L. Muller (lisam@sun.ac.za). The postal address for the Mathematics Division is:

Department of Mathematical Sciences   Tel: (021) 808-3282
Mathematics Division
University of Stellenbosch
Private Bag X1
Matieland 7602
South Africa

2 Suggested Focus Areas for the Degree

The Honours programme is flexible, and exact module choices for the second semester will be decided upon in consultation with individual students. The choice of modules should give a coherent focus to the programme, leading to opportunities for further study and employment. Suggested curricula are outlined below.

2.1 Mathematics

This focus is for students wanting a rigorous mathematics education. It consists principally of modules taught in the Mathematics Division and is usually followed by students who have a love for "pure mathematics", in particular those who intend to follow a career in research and/or teaching. (The number of credits of each module is given in brackets.)

First Semester	Second Semester
Algebra (16) Functional Analysis and Measure Theory (16) Real and Complex Analysis (16) Set Theory and Topology (16)	four 8-credit Choice Modules subject to departmental approval Honours Project (32)

2.2 Biomathematics

The Biomathematics focus aims to train students to formulate and analyse precise models for experimental data arising from real-life research problems within the fields of biology and medicine, from predicting the influence of HIV, Aids, malaria and tuberculosis to the effects of climate change on South Africa. (The number of credits of each module is given in brackets.)

First Semester	Second Semester
Computational And Discrete Methods in Biomathematics (16)	Honours project (32)
Non-linear Dynamical Systems in Biomathematics (16)	Advanced Topics in Biomathematics III (16)
Advanced Topics in Biomathematics I (8)	Advanced Topics in Biomathematics IV (8)
Advanced Topics in Biomathematics II (8)	Choice module (8)
Selected Topics from Biological Sciences (8)	
Selected Topics from Biomedical Sciences (8)	

Students registering for this focus will spend the first part of the year (January–June) at AIMS-SA (The South African centre of the African Institute for Mathematical Sciences), where they will attend a number of special modules presented by local and international specialists in modelling of biological and biomedical systems, population dynamics, biomathematics, and bio-informatics. For the second part of the year the students will be based at Stellenbosch University and involved in project work.

In the remainder of this document we give further details on modules and projects suitable for the Mathematics Focus.

3 First Semester Modules for Focus: Mathematics

The modules offered in the first semester are the core modules for the programme. Each module is worth 16 credits and is taught in three lectures per week over the semester.

3.1 Algebra (711)

The first and second quarters are dedicated to group theory and Galois theory, respectively.

In the group theory course we will introduce basic notions such as conjugates, normalisers and normal subgroups, after which we will treat various examples, such as the circle group, dihedral groups and the quaternions. (The additive group of integers modulo n and other cyclic groups are already known from 3rd year courses). Other topics include the conjugate class equation of a group, p-groups, Cauchy's Theorem and the Sylow Theorems. The Galois theory course builds upon the field theory from the 3rd year algebra course. This theory arose from investigating solutions to polynomial equations, and combines central themes from classical and modern algebra. It is closely linked with the theory of solvable groups, and some of the greatest mathematicians of the last 200 years have contributed to this subject.

Requirements: A 3rd year course in basic algebra (Mathematics 314).

Textbook: Notes will be provided.

Lecturers: Dr K.-T. Howell and Dr D. Basson.

3.2 Functional Analysis and Measure Theory (712)

The first quarter is dedicated to functional analysis and the second quarter to measure theory.

Functional Analysis: Metric and Banach spaces, bounded linear operators, functionals and dual spaces. Introduction to Hilbert spaces. The Hahn Banach theorem and its consequences, the Baire category theorem, the uniform boundedness theorem.

Measure Theory: Lebesgue outer measure, measurable set and measure, measurable functions, Littlewood's Principles. Shortcomings of the Riemann integral, the Lebesgue integral and convergence theorems. The L^p spaces.

Requirements: Third year courses in complex analysis (Mathematics 324) and in metric spaces or real analysis (Mathematics 365).

Textbooks:
Functional Analysis: E. Kreyszig: *Introductory Functional Analysis with Applications,* John Wiley & Sons Inc., New York, 1978.
Measure Theory: H. L. Royden: *Real Analysis*, Macmillan Publishing Co., Inc., New York, 1968.

Lecturers: Prof. S. Mouton and Dr R. Heymann.

3.3 Real and Complex Analysis (713)

This course is a continuation of the third year course in complex analysis and involves, among others, the following topics: Harmonic functions, Jensen's formula, Weierstrass products, the Riemann mapping theorem and the gamma and zeta functions.

Requirements: Third year courses in complex analysis (Mathematics 324) and in real analysis (Mathematics 365).

Textbook: Notes will be provided.

Lecturer: Dr S. Marques and Dr. D. Ralaivaosaona.

3.4 Set Theory and Topology (714)

In this course, each student will be required to complete assignments from one (or more) of the following areas of mathematics: axiomatic set theory (Zermelo-Frankel axioms, Zorn's lemma and the well ordering principle, cardinal and ordinal arithmetic), general topology (topology via neighborhoods, closure and interior, compactness, separation axioms, continuous functions and homeomorphisms), duality theory (lattices and Boolean algebras, Stone, Birkhoff and Priestly dualities), algebraic topology (homotopy of paths, definition and computation of fundamental group/groupoid of a topological space), and categorical topology (basic topological constructions viewed as limits and colimits in the category of topological spaces, topological functors). Students with a background in some of these areas from their undergraduate studies will be required to complete assignments that complement their background.

Requirements: A third year course in topology (Mathematics 325) or real analysis (Mathematics 365).

Textbook: Appropriate texts to be decided with individual students.

Lecturers: Prof. Z. Janelidze

4 Second Semester Choice Modules for Focus: Mathematics

We have a large list of 8-credit modules available in the second semester and each student chooses four of these (or possibly fewer if they are taking approved modules from other departments or divisions). Please note that some of the modules listed below may not be available every year. It is also possible that new modules will be added during the first semester. Students are expected to finalise their choices of second semester modules by the end of the first semester and this should be done in consultation with the Mathematics Honours coordinator and the relevant lecturers. Further information, to help with these choices, will be provided during the first semester.

Algebraic Number Theory (747)
Computational Algebra (748)
Wavelet analysis (749)
Functional Analysis II (751)
Measure Theory II (752)
Category Theory (753)
Logic (754)
Concrete Mathematics (755)
Topics in Algebra (756)
Complex Analysis II (757)
Advanced Analysis (760)
Advanced Abstract Algebra (761)
Number theory (762)
Advanced Combinatorics (767)
Algebraic Curves (768)
Algebraic Geometry (769)
Asymptotic Methods (771)
Categorical Algebra (772)
Differential Geometry (773)
Functional Analysis III (774)
Hilbert Spaces and C^*-algebras (775)
Knot Theory (776)
Lie Groups and Lie Algebras (780)
Model Theory (784)
Operator Theory (785)
Universal Algebra (781)
Representation Theory (782)
Analytic Number Theory (783)

5 Honours Project (746) for Focus: Mathematics

In the second semester students have to complete a research project on a topic of their choice. This will be evaluated through a written report and an oral presentation. The presentation takes place in the last teaching week of the second semester. The project is worth 32 credits.

Some suggested topics for projects in the Mathematics Focus are given below (for the Biomathematics Focus, students should consult the Biomathematics Convenor). The topics listed here are just suggestions. If any of them looks interesting to you, you should contact the relevant lecturer to discuss the possibility of doing that project. If you would prefer to work on a topic not listed here, you should contact a potential supervisor in the Mathematics Division to discuss your idea. Further topics may be suggested by lecturers during the first semester.

Project topics for students doing the Mathematics Focus should be selected and then approved by the Mathematics Honours Coordinator by the end of the first semester.

5.1 Feynman's fabulous formula for higher genus surfaces in the Ising model

The Ising model is a well-known statistical physics model, defined on a two-dimensional lattice. It is interesting because it exhibits a 'phase transition' at a certain critical temperature. In 1952, Feynman realized that computing the partition function of the Ising model in the plane reduces to a fascinating lemma about planar graphs — "Feynman's fabulous formula". Recently, Cimasoni has proved a higher-genus version of this formula, valid for all graphs (i.e. graphs of higher genus), and which involves spin structures on the surface. The Honours project will be about this modern topological proof of the generalized Feynman formula.

References:

- Bruce Bartlett, Feynman's fabulous formula, n-category cafè blog post. Available at: `golem.ph.utexas.edu/category/2015/06/feynmans_fabulous_formula.html`
- Dmitry Chelkak, David Cimasoni and Adrien Kassel, Revisiting the combinatorics of the 2d Ising model. Available at: `arxiv.org/abs/1507.08242`.

Project Supervisor: Dr B. Bartlett

5.2 Approximating Quadratic Algebraic Numbers

In Diophantine approximation one studies approximations of real numbers α by rational numbers $\frac{p}{q}$. Since the rationals are dense in the reals, the value $\left|\alpha - \frac{p}{q}\right|$ can always be decreased. But one would like to study by how much you need to increase q in order to obtain an improvement. Therefore mathematicians decided to ask for constants c and n such that

$$\left|\alpha - \frac{p}{q}\right| < c \cdot q^{-n}.$$

The smaller we choose c and the larger we choose n the fewer solutions p, q exist. So, one might ask at what point we change from having infinitely many solutions to finitely many.

There are many high powered theorems in this direction, but in this project I want the student to focus on the case where α is a quadratic algebraic number. In that case it is a theorem that there exists α such that there are infinitely many p, q satisfying

$$\left|\alpha - \frac{p}{q}\right| < \frac{1}{\sqrt{5}q^2}.$$

However, for most α there are only finitely many. In the project the student will investigate for which numbers α this inequality has infinitely many solutions and what possible better bounds exist if we remove those possibilities for α.

Project Supervisor: Dr D. Basson

5.3 Model theory of algebraically closed fields and the Ax-Grothendieck theorem

Algebraically closed fields (such as the field of complex numbers) are very well-behaved from the point of view of model theory. The main results are that the first-order theory of algebraically closed fields (in the language of rings) has quantifier elimination and, for any p which is prime or zero, the first-order theory of algebraically closed fields of characteristic p is complete. The main aim of this essay is to present these results and show how they are used to prove the Ax-Grothendieck theorem which asserts that every injective polynomial map from $\mathbb{C}^n$ to $\mathbb{C}^n$ is surjective. There is scope to take the ideas further and consider other applications.

Prerequisite: Some background in model theory (such as Mathematics 345).

Project Supervisor: Dr G. Boxall

5.4 Categorical Mathematics

Depending on the interest of the student, this project will study a topic in classical mathematics from the category-theoretic perspective.

Recommended module: Category Theory

Project Supervisor: Dr J. Gray

5.5 Mathematical structures with real-life origins

The aim of this project is to study various structures in abstract mathematics which can be discovered from ideas arising directly from the real-life phenomena or activities. This project should be particularly interesting for those who are thinking of a teaching career in mathematics.

Project Supervisor: Prof. Z. Janelidze

5.6 Examples of functors

A function assigns to elements of one set, element of another set. A functor assigns to mathematical structures and structure-preserving maps of a given type, mathematical structures and structure-preserving maps of another type. The aim of this project is to look at important functors encountered in different branches of mathematics, such as linear algebra, abstract algebra, group theory, topology, algebraic topology, logic, etc.

Project Supervisor: Prof. Z. Janelidze

5.7 Matroids

A *matroid* is an abstract combinatorial structure. Particular kinds of matroids arise from graphs (in many ways), from so-called submodular set functions (i.e. satisfying $f(X \cup Y) + f(X \cap Y) \leq f(X) + f(Y)$), from the columns of any matrix (by focusing on the linear dependency relations among them), from certain types of lattices (i.e. simultaneously atomistic and semimodular ones), and from many other scenarios. What is more, even the abstract concept 'matroid' can be defined in a stunning number of equivalent ways. For instance a matroid can be defined as a closure operator satisfying a certain exchange property, as a family of sets (called circuits) satisfying certain circuit-axioms, as a rank function on a powerset satisfying certain rank-axioms, and so on.

Project Supervisor: Prof. M. Wild

5.8 Finite dimensional near-vector space constructions using copies of a finite field

Near-vector spaces differ from traditional vector spaces in that they possess less linearity. In this project we will investigate how finite field theory is used in the construction of all finite dimensional near-vector spaces constructed using copies of a given finite field. We will focus on the near-vector spaces first defined by André.

Project Supervisor: Dr K.-T. Howell