Pricing and Hedging of Options using Numerical methods

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1. Introduction

2. The Model

3. The Hedging factors


5. Conclusion and Present work
Introduction

An option is a written agreement between two parties, the holder (*buyer*) of an option and the writer (*seller*) of an option. For the first time, in 1973, options were traded on stock in an organised exchange. There are two basic types of Option. A *call* and a *put* option. We have the path independent (European option) and the path dependent options. The payoff of European call is

\[(S_T - K)^+ = \begin{cases} S_T - K & \text{if } S_T \geq K, \\ 0 & \text{else} \end{cases}\]

And payoff of European put is given by

\[(K - S_T)^+ = \begin{cases} K - S_T & \text{if } S_T \leq K, \\ 0 & \text{else} \end{cases}\]
In 1973, *Fischer Black and Myron Scholes* articulated the popularly known *Black Scholes model*. The model was based on some assumptions.

**Definition**

\[ W = (W_t, t \geq 0) \text{ is a real-valued Brownian motion starting from 0 on } (\Omega, \mathcal{F}, \mathbb{P}) \text{ if} \]

(a) \( \mathbb{P}(W_0 = 0) = 1 \),

(b) *For all* \( 0 \leq s \leq t \), the real valued random variable \( W_t - W_s \) *is normally distributed with mean 0 and variance* \( t - s \).

(c) *For all* \( 0 = t_0 < t_1 < \cdots < t_p = T \), the variable \( (W_{t_k} - W_{t_{k-1}}, 1 \leq k \leq p) \) *are independent.*
Theorem (Risk Neutral Valuation)

Let $\phi$ be a replicating portfolio for an option $C$, i.e. if $V_T(\phi) = C_T$, then we must have $V_0(\phi) = C_0$ or else there will be arbitrage. (if $V_0(\phi) > C_0$), then you buy the cheaper $C$ and sell the more expensive $\phi$. The difference $V_0(\phi) - C_0$ is free. And at expiry $T$ the portfolio and the option cancel out. Thus, you have made a certain positive gain of $V_0(\phi) - C_0$ at no cost: There is arbitrage! But then

$$V_0(\phi) = C_0 = \mathbb{E}_Q[\bar{V}_T(\phi)] = \mathbb{E}_Q[\bar{C}_T]$$

because $\bar{V}_t(\phi)$ is a $Q-$ martingale, and thus

$$C_0 = \mathbb{E}_Q[e^{-rT}C_T].$$
Theorem (Black-Scholes Equation)

We want to price a contingent claim of the form $\phi(S_T)$. Having known that the model satisfy the Risk neutral valuation theorem and $S_T$ is a stochastic variable, we implement the 1-dimensional Itô’s Formula and the hedging strategy. Thus the Black Scholes differential equation is given as:

$$\frac{\partial f}{\partial t} + rS \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} - rf = 0 \quad BSDE.$$ 

Thus; $BS(S_0, \sigma, T, r, K) = S_0 N \left( \frac{\log(S_0/K) + (r + \sigma^2/2) T}{\sigma \sqrt{T}} \right) - e^{-rT} KN \left( \frac{\log(S_0/K) + (r - \sigma^2/2) T}{\sigma \sqrt{T}} \right)$
The Greeks

Greeks are price sensitivities which are to quantify the risk exposure of a financial derivative investment. For instance, the five popularly used Greeks are,
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![Graph showing Delta for a European call option.](image)
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![Delta for a European call option](image1.png)

![Gamma for a European call option](image2.png)
Monte Carlo were first applied to pricing in 1977, by Phelim Boyle. The act of evaluating $f$ at $M$ (where $M$ is a fixed value) of random points and averaging the results is the Monte Carlo estimate.
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\[ \hat{C}_0 = \sum_{j=1}^{M} \frac{1}{M} \exp(-rT)(S_{T,j} - K)^+. \] (1)

The standard estimator for the variance of \( C_0 \) is given by

\[ \hat{\mathcal{V}}(C_{0,j}) = \sqrt{\frac{1}{M-1} \sum_{j=1}^{M} (C_{0,j} - \hat{C}_0)^2} \]

A measure of the standard mean error of the sample mean \( \hat{C}_0 \) is given by

\[ S_M(\hat{C}_0) = \frac{\hat{SD}(C_{0,j})}{\sqrt{M}} \] (2)

where \( \hat{SD}(C_{0,j}) = \sqrt{\hat{\mathcal{V}}(C_{0,j})} \) is the standard deviation of \( \hat{C}_0 \). A confidence interval for example, a 95 percent confidence interval of \( \hat{C}_0 \), is an interval \([L, U]\) with random endpoints. The probability \( P[L \leq \hat{C}_0 \leq U] = 0.95 \).
We can approximately $N(\mu_C, \sigma_C^2)$ random variable $C$, use the approximation,

$$P [\mu_C - 1.96\sigma_C \leq x \leq \mu_C + 1.96\sigma_C] \approx 0.95.$$ 

Therefore the confidence interval is given by

$$\left[ \hat{C}_0 - 1.96 \frac{\hat{SD}(C_{0,j})}{\sqrt{M}}, \hat{C}_0 + 1.96 \frac{\hat{SD}(C_{0,j})}{\sqrt{M}} \right].$$  \hspace{1cm} (3)

In all numerical experiments, $\sigma = 0.7$, $r = 0.05$ and $T = 1$. Otherwise, the value of the parameter will be given.

<table>
<thead>
<tr>
<th>Strike (K)</th>
<th>Black-Scholes</th>
<th>Monte Carlo</th>
<th>Variance</th>
<th>Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>85</td>
<td>34.2359</td>
<td>34.2733</td>
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<td>26.5677</td>
<td>3900.61</td>
<td>[25.3422,27.7033]</td>
</tr>
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</table>
Variance Reduction techniques

To avoid the unpredictable amount of experiment required, it is profitable to distort the original problem in such a way to reduce the uncertainty in the result.

This brought up the idea of variance reduction techniques which measure the uncertainty in term of variance. These techniques have made Monte Carlo methods efficient in area of research where the methods has failed before. These techniques are control variate, antithetic variate, stratified sampling and important sampling.

\[
\text{Efficiency} = \frac{\text{Variance of Monte Carlo method}}{\text{Variance of new estimator}}
\]
The idea of control variates is to find samples that have some general known correlation. The control variate is $\hat{Y}_{cv} = \hat{Y} - \beta^*(X - E_x)$. In the Black-Scholes model for European call options,

$$\hat{C}(S_0) = \frac{1}{n} \sum_{i=1}^{n} (\phi(S_T^i) - \beta^*(S_T^i - S_0 e^{rT}))$$
The idea of control variates is to find samples that have some general known correlation. The control variate is \( \hat{Y}_{cv} = \hat{Y} - \beta^*(X - \mathbb{E}_X) \). In the Black-Scholes model for European call options,

\[
\hat{C}(S_0) = \frac{1}{n} \sum_{i=1}^{n} \left( \phi(S^i_T) - \beta^*(S^i_T - S_0 e^{rT}) \right)
\]

<table>
<thead>
<tr>
<th>Strike (K)</th>
<th>Monte Carlo</th>
<th>Control Variate</th>
<th>Efficiency</th>
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<td>Confidence Interval</td>
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<td>[27.0231 , 35.3572]</td>
<td>588.48</td>
</tr>
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</table>

Table: \( M = 1000 \).
Antithetic variate

This method attempts to reduce variance by generating averages from samples which have negative covariance between them. An antithetic path is

\[ \hat{I}_A = \frac{1}{M} \sum_{i=1}^{M} \frac{f(Z_i) + f(-Z_i)}{2} \]
Monte Carlo Methods
The Fourier cosine series expansion method.

<table>
<thead>
<tr>
<th>Strike (K)</th>
<th>Monte Carlo Variance</th>
<th>Confidence Interval</th>
<th>Antithetic Variate Variance</th>
<th>Confidence Interval</th>
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<td>32.0397</td>
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<td>31.3512</td>
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<td>28.0958</td>
<td>27.7469</td>
<td>29.8274</td>
<td>28.6677</td>
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Estimation of the Greeks

Estimating sensitivities of expectation of the Greeks is an important strategy that requires deep understanding. Prices can be known in the market directly but derivatives of price cannot be observed directly. We will consider three methods and these are;

\[ \Delta = \frac{\partial}{\partial S} E_Q \left[ e^{-rT} (S_T - K) + \right] = V(\phi(S_T + h)) - V(\phi(S_T - h)) \]

\[ \Gamma = V(\phi(S_T + h)) - 2V(\phi) + V(\phi(S_T - h)) \]

where \( V(\phi(S_T)) = E_Q \left[ e^{-rT} (S_T - K) + \right] \) and \( h \) is a constant.
Estimation of the Greeks

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- Finite difference method:

\[
\Delta = \frac{\partial}{\partial S_0} \mathbb{E}_Q[e^{-rT}(S_T - K)^+] = \frac{V(\phi(S_T + h)) - V(\phi(S_T - h))}{2h}
\]

\[
\Gamma = \frac{V(\phi(S_T + h)) - 2V(\phi) + V(\phi(S_T - h))}{h^2}
\]

where \( V(\phi(S_T)) = \mathbb{E}_Q[e^{-rT}(S_T - K)^+] \) and \( h \) is a constant.
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\]

where \( V(\phi(S_T)) = \mathbb{E}_Q[e^{-rT}(S_T - K)^+] \) and \( h \) is a constant.
Pathwise derivative:

\[ \Delta = \frac{d\phi}{dS_0} = \frac{d\phi}{dS_T} \times \frac{dS_T}{dS_0} = e^{-rT} \frac{S_T}{S_0} \mathbb{I}_{S_T > K}. \]
Pathwise derivative:
\[ \Delta = \frac{d\phi}{dS_0} = \frac{d\phi}{dS_T} \times \frac{dS_T}{dS_0} \]
\[ = e^{-rT} \frac{S_T}{S_0} \mathbb{I}_{S_T > K}. \]

Likelihood ratio
\[ \Delta = \mathbb{E} \left[ e^{-rT} \phi(S_T) \frac{Z}{S_0 \sigma T} \right]. \]
\[ \Gamma = \frac{N(S_T)^2 - 1}{S_0^2 \sigma^2 T} - \frac{N(S_T)^2}{S_0^2 \sigma^2 \sqrt{T}}. \]
Numerical Results of the Estimation of Greeks

Delta of European call option

Gamma of European call option

Monte Carlo Methods
The Fourier cosine series expansion method.

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<tr>
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<th>Finite difference</th>
<th>Pathwise derivative</th>
<th>Likelihood ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>0.9929</td>
<td>0.9941(0.1291)</td>
<td>0.9900(0.1386)</td>
<td>1.0148(2.6635)</td>
</tr>
<tr>
<td>90</td>
<td>0.8870</td>
<td>0.8857(0.3564)</td>
<td>0.8896(0.3516)</td>
<td>0.8903(1.8545)</td>
</tr>
<tr>
<td>100</td>
<td>0.5399</td>
<td>0.5437(0.5394)</td>
<td>0.5448(0.5395)</td>
<td>0.5456(1.2536)</td>
</tr>
<tr>
<td>110</td>
<td>0.1829</td>
<td>0.1833(0.4216)</td>
<td>0.1786(0.4174)</td>
<td>0.1868(0.7193)</td>
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<table>
<thead>
<tr>
<th>Strike (K)</th>
<th>Black-Scholes</th>
<th>Finite difference</th>
<th>Likelihood ratio</th>
</tr>
</thead>
<tbody>
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<td>0.0021</td>
<td>0.0241(3.1183)</td>
<td>0.0023(0.1945)</td>
</tr>
<tr>
<td>90</td>
<td>0.0202</td>
<td>0.0108(2.8780)</td>
<td>0.0181(0.1147)</td>
</tr>
<tr>
<td>100</td>
<td>0.0418</td>
<td>0.0111(2.0228)</td>
<td>0.0393(0.0555)</td>
</tr>
<tr>
<td>110</td>
<td>0.0279</td>
<td>0.0321(0.9849)</td>
<td>0.0288(0.0248)</td>
</tr>
</tbody>
</table>

**Table:** The estimated value of the Delta and Gamma is presented with the standard deviation (in bracket) with $M = 10000$. 
Definition (Fourier cosine series expansion)

Let \( f(\theta) \) be a function defined and integrable on the interval \((-\pi, \pi)\), the Fourier series of \( f(\theta) \) is given by

\[
f(\theta) = \frac{A_0}{2} + \sum (A_n \cos(\theta n) + B_n \sin(\theta n))
\]

with

\[
A_n = \frac{2}{\pi} \int_{-\pi}^{\pi} f(\theta) \cos(n\theta) d\theta
\]

Recall the Risk neutral valuation formula

\[
C(x, t) = e^{-r\Delta t} \mathbb{E}^{a,b}_t [C(y, T)|x]
\]

\[
= e^{-r\Delta t} \int_{a}^{b} C(y, T)f(y|x)dy
\]
The series expansion of $f(y|x)$ with

$$A_n(x) = \frac{2}{b-a} \int_a^b f(y|x) \text{Re}\{e^{in\pi \frac{y-a}{b-a}}\} dy = \frac{2}{b-a} \text{Re}\{e^{-in\pi \frac{a}{b-a}} \phi \left( \frac{n\pi}{b-a} ; x \right) \}$$

$$C_n = \frac{2}{b-a} \int_a^b C(y, T) \cos(n\pi \frac{y-a}{b-a}) dy,$$ where $C(y, T) = [\alpha K(e^y - 1)]^+$

And $y = \ln(S_T/K)$, $x = \ln(S_0/K)$.

$$C(x, t_0) = \frac{b-a}{2} e^{-r\Delta t} \sum_{n=0}^{N-1} \text{Re}\{e^{-in\pi \frac{a}{b-a}} \phi \left( \frac{n\pi}{b-a} ; x \right) \}. C_n$$ (8)

Equation (8) is the COS formula for general underlying processes.
The Greeks

- **Delta:**

\[ \Delta = \frac{\partial C(x, t_0)}{\partial S_0} = \frac{\partial C(x, t_0)}{\partial x} \frac{\partial x}{\partial S_0} \]  

(9)

Note: \( x = \log\left(\frac{S_0}{k}\right), \)

\[ \Delta = \frac{1}{S_0} \frac{\partial C(x, t_0)}{\partial x} \]  

(10)

- **Gamma:**

\[ \Gamma = \frac{\partial^2 C(x, t_0)}{\partial S_0^2} \]  

(11)
Simulations

Here we presents the unknown parameters in equation 8

\[
\phi_x(t) = \int_{\mathbb{R}} \exp(itx) \frac{1}{\sqrt{(2\pi\sigma^2)}} \exp \left( \frac{-(x - \mu)^2}{2\sigma^2} \right) dx = \exp(i\mu t - \frac{1}{2}\sigma^2 t^2)
\]

(12)

The Moment generating function \( M_x(t) \) is;

\[
M_x(t) = \phi(-i, t) = \exp \left[ (-i(i\mu t)) - \frac{(-i)^2(\sigma^2 t^2)}{2} \right] = \exp \left( \mu t + \frac{\sigma^2 t^2}{2} \right).
\]

The cumulant generating function that is the log of the Moment generating function is given by;

\[
\psi_x(t) = \log(\exp(\mu t + \frac{\sigma^2 t^2}{2})) = \mu t + \frac{\sigma^2 t^2}{2}.
\]

Thus, the cumulants are the derivative of the \( \psi(t) \) at \( t = 0 \) is given by;
\begin{align*}
  c_1 &= \mu T \quad c_2 = \sigma^2 T \quad c_3 = 0 \quad c_4 = 0. \\
\end{align*}

The truncated range of integration \( a \) and \( b \) can be written as

\[ [a, b] := [c_1 - L \sqrt{c_2 + c_4}, c_1 + L \sqrt{c_2 + c_4}] \quad \text{with} \quad L = 10 \]

Thus, equation 14 becomes;

\[ [a, b] := [\mu T - L \sqrt{\sigma^2 T}, \mu T + L \sqrt{\sigma^2 T}] \]
Figure: Short-dated ($T = 0.1$) European calls under the Black-Scholes using the Cos method and the Exact formula. Parameters used. $K \in [0, 400]$, $N = 64$, $S_0 = 100$, $T = 0.1$, $\sigma = 0.25$, $r = 0.1$
The Greeks using Cos methods

Figure: Delta and Gamma of European calls under the Black-Scholes using the Cos method and the Exact formula. Parameters used. $K \in [0, 400]$, $N = 64$, $S_0 = 100$, $T = 0.1$, $\sigma = 0.25$, $r = 0.1$
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## Convergence error

<table>
<thead>
<tr>
<th>Strike (K)</th>
<th>Black-Scholes</th>
<th>Cos method</th>
<th>Pathwise derivative(MC)</th>
</tr>
</thead>
<tbody>
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<td>0.9929</td>
<td>0.9929(1.110e-16)</td>
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<tr>
<td>90</td>
<td>0.8870</td>
<td>0.8857(1.110e-16)</td>
<td>0.8896(0.0026)</td>
</tr>
<tr>
<td>100</td>
<td>0.5399</td>
<td>0.5399(2.220e-16)</td>
<td>0.5448(0.0049)</td>
</tr>
<tr>
<td>110</td>
<td>0.1829</td>
<td>0.1829(4.996e-16)</td>
<td>0.1786(0.0043)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Strike (K)</th>
<th>Black-Scholes</th>
<th>Cos method</th>
<th>Likelihood ratio (MC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>0.0021</td>
<td>0.0021(7.097e-09)</td>
<td>0.0023(0.0006)</td>
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<td>90</td>
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<td>0.0181(0.0179)</td>
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<td>0.0393(0.0025)</td>
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<tr>
<td>110</td>
<td>0.0279</td>
<td>0.0279(8.179e-07)</td>
<td>0.0288(0.0009)</td>
</tr>
</tbody>
</table>

**Table:** The Delta and the Gamma, (in bracket) is the absolute error, for MC $M = 10000$, the Cos method $N = 64$ and $S_0 = 100$. 

Fadina Tolulope Rhoda
Pricing and Hedging of Options using Numerical methods
In computational finance, accuracy and speed plays a vital role.

- For pricing of European call option with the Monte Carlo methods, the control variate method is the best.
- The pathwise derivative provides the best estimate for the Delta and the likelihood ratio show the best estimate for the gamma.
- Comparing the two methods, the Cos method was preferred to the Monte Carlo methods, not only in term of efficiency but the fact that a large range of strike price can be priced at a goal.

Presently, we are doing two things at the same time. Firstly, we are at the final stage of pricing European call options in the Lévy model (Variance Gamma model) using the Cos method. Secondly, we are developing an algorithm for pricing more complicated options in the Black-Scholes model and Lévy model.